Casualty Actuarial Society examination questions

Questions from actuarial society examinations

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This chapter works questions from actuarial society examinations. Each section names the examination sitting and the question number. The first question comes from the Casualty Actuarial Society's Exam 9 and applies the percentile layer capital allocation of Bodoff (2007), which the monograph, *Working with Aggregate Probability Distributions*, treats at length in its chapter on Bodoff's method.

1 Exam 9, Spring 2018, Question 15

An insurer has exposure to two independent perils, wind and earthquake:

- Wind has a 15% chance of a $5 million loss, and an 85% chance of no loss.
- Earthquake has a 1 % chance of a $15 million loss, and a 99% chance of no loss.

Using the capital allocation by percentile layer methodology with a 99.5% VaR capital requirement, determine how much capital should be allocated to each peril.

Solution.

The last row gives the percentile layer capital.

```
from aggregate import qd
from aggregate.pedagogy import bodoff_exhibit

cas15 = build('port CASq15 '
     'agg X1 1 claim dsev [0,  5] [0.85, 0.15] fixed '
     'agg X2 1 claim dsev [0, 15] [0.99, 0.01] fixed ')
qd(cas15)
# cas15.update(bs=1, log2=8, remove_fuzz=True, padding=1)
cas15.density_df = cas15.density_df.apply(lambda x: np.round(x, 10))
basic = bodoff_exhibit(cas15, reg_p=.995)
qd(basic, col_space=10)
df = cas15.density_df.query('S > 0 or p_total > 0')
```

```
Portfolio object: CASq15. Contains 2 aggregate components. Updated with
bucket size 1 and log2 = 11. Validation: not unreasonable.

            Mean     SD     CV   Skew P01 Median P99
unit  X
X1    Freq     1      0      0
      Sev   0.75 1.7854 2.3805 1.9604   0      0   5
      Agg   0.75 1.7854 2.3805 1.9604   0      0   5
X2    Freq     1      0      0
      Sev   0.15 1.4925 9.9499 9.8494   0      0   0
```

```
      Agg   0.15 1.4925 9.9499 9.8494   0      0   0
total Agg    0.9  2.327 2.5856 3.4839   0      0  15
```

method	X1	X2	total
EX	0.75	0.15	0.9
sa VaR	5	15	15
sa TVaR	5	15	16.5
pct EX	12.5	2.5	15
coVaR	-0	15	15
alt coVaR	0.5625	14.438	15
naive coTVaR	0.71429	14.286	15
coTVaR	0.75	15	15.75
plc	5.0714	9.9286	15

The calculation of `plc` as the integral of α for unit 1 is simply:

```
df.exi_xgta_X1.shift(1, fill_value=0).cumsum().loc[15] * cas15.bs
```

```
np.float64(5.071372239499997)
```

Bodoff, N. M. (2007). Capital Allocation by Percentile Layer. *Variance*, *3*(1), 13–30.